Non-Dog is my Co-Pilot
Departures from Gaussianity in Large Scale Structure

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With Amber Miller, Sarah Shandera and Licia Verde  arXiv: 0711.4126
Outline

- Why look for (scale dependent) non-Gaussianity?
- Where to look for (scale-dependent) non-Gaussianity?
  - Cluster counts: deriving the non-Gaussian mass function
  - Cluster counts: the non-Gaussian mass function
  - The bispectrum
- Conclusion
Motivation and Review of Initial Conditions
Do we have a compelling model of the early universe?

- Inflation solves: homogeneity, flatness, formation of structure
- What viable models of inflation are there?
- How can we distinguish between them?
Inflation predicts

• flat, homogeneous universe
• almost scale invariant, nearly Gaussian, adiabatic density perturbations
• some tensor perturbations
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Inflation predicts:

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- some tensor perturbations

Some single field models predict large non-Gaussianity
Gaussian?

For a single variable this is easy

\[ P(x)dx = \frac{dx}{\sqrt{2\pi}\sigma^2} e^{-x^2/2\sigma^2} \]

\[ \langle f(x) \rangle = \int_{-\infty}^{\infty} dx \, f(x) P(x) \]
Gaussian?

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\[
\begin{align*}
\langle x \rangle &= 0 \\
\langle x^2 \rangle &= \sigma^2 \\
\langle x^3 \rangle &= 0 \\
\langle x^4 \rangle &= 3\sigma^4 \\
\cdots
\end{align*}
\]
Gaussian?

For a field we have to do more work

$$P(\phi_1, \phi_2, \ldots, \phi_n) = \frac{1}{(2\pi)^{n/2} \sqrt{\text{det } \sigma}} e^{-\frac{1}{2} \phi_i \sigma_{ij}^{-1} \phi_j}$$

still only quadratic in fields

but correlation between points allowed
Gaussian?

For a field we have to do more work

\[ P(\phi_1, \phi_2, \ldots, \phi_n) = \frac{1}{(2\pi)^{n/2} \sqrt{\det \sigma}} e^{-\frac{1}{2} \phi_i \sigma_{ij}^{-1} \phi_j} \]

\[ \langle \phi_i \phi_j \rangle = \sigma_{ij} \]
\[ \langle \phi_i \phi_j \phi_k \rangle = 0 \]
\[ \langle \phi_i \phi_j \phi_k \phi_l \rangle = \sigma_{ij} \sigma_{kl} + \sigma_{ik} \sigma_{jl} + \sigma_{il} \sigma_{jk} \]
\[ \langle \phi_i \phi_j \phi_k \phi_l \phi_m \rangle = 0 \]
\[ \ldots \]
Gaussian?

in the continuum limit

\[ P[\phi] = \frac{e^{-\int \int d^3 x \phi(x) \sigma(|x-y|) \phi(y)}}{\int D\phi e^{-\int \int d^3 x \phi(x) \sigma(|x-y|) \phi(y)}} \]

or in Fourier space

\[ P[\phi] = \frac{e^{-\int \frac{d^3 k}{(2\pi)^3} \tilde{\phi}(k) \tilde{\sigma}(|k|) \tilde{\phi}^*(k)}}{\int D\phi e^{-\int \frac{d^3 k}{(2\pi)^3} \phi(k) \sigma(|k|) \phi^*(k)}} \]
Gaussian?
in the continuum limit

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independent of the phase
An example

A “local” model

\[ \phi_{NG} = \phi_G + f_{NL}(\phi_G^2 - \langle \phi_G^2 \rangle) + \ldots \]

- \( \phi_G \) Gaussian
- \( f_{NL} \) constant so that mean and variance of \( \phi_G \) and \( \phi_{NG} \) agree (to \( O(f_{NL}^2 \phi_G^2) \))
- the three point of \( \phi_{NG} \), \( \langle \phi_{NG} \phi_{NG} \phi_{NG} \rangle \propto f_{NL} \)
- not completely general
What can we learn about inflation?

power spectrum:

\[ \langle \zeta(k)\zeta(k') \rangle = (2\pi)^3 \delta(k + k') \frac{2\pi^2 \mathcal{P}(k)}{k^3} \]

\[ \mathcal{P}(k) \propto k^{n_s - 1} \]

primordial curvature

\[ \delta(k, a) \propto +\zeta(k) \]
What can we learn about inflation?

Power spectrum:

\[
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\]

\[\mathcal{P}(k) \propto k^{n_s - 1}\]

\[n_s - 1 = -2\epsilon - \eta = \text{(slow roll parameters, depend on inflationary potential)}\]
What can we learn about inflation?

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$$P(k) \propto k^{n_s - 1}$$

also have tensor power spectrum:

$$P_h \propto k^{n_T}$$
What can we learn about inflation?

Power spectrum:

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\[ P(k) \propto k^{n_s - 1} \]

Also have tensor power spectrum:

\[ P_T \propto k^{n_T} \]

tensor spectral index \(= -2 \varepsilon\)

Amplitude \(\sim H^2\) during inflation
What can we learn about inflation?

power spectrum:

$$\langle \zeta(k)\zeta(k') \rangle = (2\pi)^3 \delta(k + k') \frac{2\pi^2 P(k)}{k^3}$$

also have tensor power spectrum:

$$P_T(k) \propto k^{n_T}$$

and the tensor to scalar ratio

$$r = \frac{P_T}{P_s} = 16 \varepsilon$$
What can we learn about inflation?

Summary:

amplitude of scalar perturbations, $A_0 \propto H^2/\varepsilon$
scalar spectral index, $n_s = -2\varepsilon - \eta$
tensor to scalar ratio, $r = 16\varepsilon$
tensor spectral index, $n_T = -2\varepsilon$
Motivation: the DBI model
Motivation: the DBI model

“speed limit”

\[ \gamma(\phi) = \frac{1}{\sqrt{1 - \dot{\phi}^2 T^{-1}}} \]

sound speed

\[ c_s = \frac{1}{\gamma} \]

slow roll equations modified \( c \longrightarrow c_s \)
What can we learn? (about sound speed inflation)

Summary:

- amplitude of scalar perturbations, \( A_0 \propto H^2/c_s \varepsilon \)
- scalar spectral index, \( n_s \left( =-2\varepsilon-\eta-\kappa \right) \)
- tensor to scalar ratio, \( r \left( =16\varepsilon c_s \right) \)
- tensor spectral index, \( n_T \left( =-2\varepsilon \right) \)

Factors of the sound speed \( c_s \) have appeared

new slow roll parameter is added

\[ \kappa = \frac{\dot{c}_s}{Hc_s} \]
The skewness in arbitrary sound speed models

\[ \langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle = (2\pi)^7 \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \frac{\mathcal{P}(K)^2}{k_1^3k_2^3k_3^3} (\mathcal{A}_\lambda + \mathcal{A}_c + \mathcal{A}_o + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s) \]

\[ \langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) 2\pi^2 \mathcal{P}(k) k^{-3} \]

Chen, Huang, Kachru, Shiu (hep-th/0605045)
The skewness in arbitrary sound speed models

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1+k_2+k_3) \frac{P(K)^2}{k_1^3 k_2^3 k_3^3} (A_\lambda + A_c + A_o + A_\epsilon + A_\eta + A_s) \]

\[ \langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) 2\pi^2 P(k) k^{-3} \]

primordial curvature \[ \delta(k, a) \propto + \zeta(k) \]

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The skewness in arbitrary sound speed models

$$K = k_1 + k_2 + k_3$$

$$\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) \frac{P(K)^2}{k_1^3 k_2^3 k_3^3} (A_{\lambda} + A_c + A_o + A_e + A_\eta + A_s)$$

model dependent functions of $k_1$, $k_2$, $k_3$

proportional to slow roll parameters

Chen, Huang, Kachru, Shiu (hep-th/0605045)
The skewness in arbitrary sound speed models

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1+k_2+k_3) \frac{P(K)^2}{k_1^3k_2^3k_3^3} (A_\lambda + A_c + A_o + A_\epsilon + A_\eta + A_s) \]

K = k_1 + k_2 + k_3

Model dependent functions of k_1, k_2, k_3

Proportional to slow roll parameters

Vanishes for DBI

Chen, Huang, Kachru, Shiu (hep-th/0605045)
The skewness in arbitrary sound speed models

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1+k_2+k_3) \frac{P(K)^2}{k_1^3 k_2^3 k_3^3} (A_\lambda - A_c + A_o + A_\epsilon + A_\eta + A_s) \]

K = k_1 + k_2 + k_3

proportional to slow roll parameters

vanishes for DBI

dominant term

model dependent functions of k_1, k_2, k_3

Chen, Huang, Kachru, Shiu (hep-th/0605045)
Our starting point

\[
\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) P^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3}
\]

\[
A_c(k_1, k_2, k_3) \propto - \left( \frac{1}{c_s^2(K)} - 1 \right)
\]

\[
c_s(k) \propto k^\kappa
\]
Our starting point “DBI”

\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) P^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3}
\]

\[
A_c(k_1, k_2, k_3) \propto - \left( \frac{1}{c_s^2(K)} - 1 \right)
\]

\[c_s(k) \propto k^\kappa\]

\(\kappa < 0\), slow roll parameter
Our starting point

“equilateral”

\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_1) \mathcal{P}_\zeta(K) \frac{A_{\text{equil.}}(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3}
\]

\[A_{\text{equil.}} \propto f_{\text{NL}}^{\text{equil.}}(k)\]

\[f_{\text{NL}}^{\text{equil.}} \propto k^{-2\kappa}\]

Creminelli et al astro-ph/0509029
The shape?

\[ \langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) \mathcal{P}^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3} \]

maximal for equilateral geometry

Babich, Creminelli, Zaldarriaga astro-ph/0405356
The running?

\[ \left\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \right\rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) P^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3} \]

If

\[ a^{6 + 2(n_s - 1) - 2\kappa} P^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1 k_2 k_3} \]

Babich, Creminelli, Zaldarriaga astro-ph/0405356
How does this compare to $f_{NL}$ (local) models?

$$\zeta_{NG} = \zeta_G + \frac{3}{5} f_{NL} (\zeta_G^2 - \langle \zeta_G^2 \rangle)$$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) (2\pi^2)^2 \left[ \frac{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2)}{k_1^3 k_2^3} + \text{(perms)} \right]$$

maximal for squeezed triangles

Babich et al astro-ph/0405356
How does this compare to $f_{NL}$ (local) models?

$$\zeta_{NG} = \zeta_{G} + \frac{3}{5} f_{NL} (\zeta_{G}^2 - \langle \zeta_{G}^2 \rangle)$$

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3) (2\pi^2)^2 \left[ \frac{P_{\zeta}(k_1)P_{\zeta}(k_2)}{k_1^3 k_2^3} + (perms) \right]$$

scales as $a^{-6+2(n_s-1)}$
How does this compare to $f_{\text{NL}}$ (local) models?

geometry dependence is different

sound speed:

$$k_1 \triangleleft k_2$$

$$k_3$$

scale dependence is different

sound speed:

$$a^{-6+2(n_s-1)-2\kappa}$$

$$f_{\text{NL}}$$

$$k_3 \rightarrow k_1$$

$$k_2$$

Babich et al astro-ph/0405356
How does this compare to $f_{NL}$ models?

\[ A_c(1, k_2, k_3)/(k_2 k_3)/f_{NL} \quad \text{and} \quad A_{local}(1, k_2, k_3)/(k_2 k_3)/f_{NL} \]
FYI: slow roll inflation predicts

Local shape:

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle \]

very small amplitude \[ f_{NL} \propto (n_s - 1) \]

Detection of large primordial non-Gaussianity puts slow roll inflation in jeopardy
Constraining non-Gaussianity

Scale dependent non-Gaussianity

\[ "f_{NL}^{\text{DBI}}" = -\frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right) \]

\[ f_{NL} \propto k^{0.2} \]

\[ f_{NL} \propto k^{0.6} \]
(II)

Constraints from Clusters
how do clusters know about non-Gaussianity?

Clusters form when the (smoothed) density fluctuation is above a threshold $\delta_c$
how do clusters know about non-Gaussianity?

positive skewness

more collapsed objects
how do clusters know about non-Gaussianity?

- positive skewness: more collapsed objects
- negative skewness: fewer collapsed objects
how do clusters know about non-Gaussianity?

positive skewness
  → more collapsed objects

negative skewness
  → fewer collapsed objects

The number of clusters tells us about the tail of the probability distribution function

\[ P(\delta)\,d\delta \]
Cluster counts

\[ P(\delta_R) d\delta_R \]

\[ n(M) dM \] – number density of clusters with mass between \( M \) and \( M + dM \)

\[ dN/dz \] – number of clusters per redshift interval \( dz \) with mass bigger than \( M_{\text{lim}} \)
Recipe for $n(M)$

- Recover the probability distribution function (PDF) from $\langle \delta^3(\mathbf{R}) \rangle$
- Follow Press–Schechter approach

\[
f_{\text{collapse}}(M) = 2 \int_{\delta_c}^{\infty} d\delta P(\delta, M)
\]

\[
\frac{dn}{dM} = -\frac{\bar{\rho}}{M} \frac{df}{dM}
\]

- Use $n_{\text{NG}}(M)/n_{\text{G}}(M)$ where each is found from above

Press & Schechter 1974  
Robinson & Baker astro-ph/9905098  
Mattarese, Verde, Jimenez astro-ph/0001366
Caveats/Difficulties

- We recover an approximate p.d.f limited range (in mass and redshift) of validity.
- This approach will indicate the utility of cluster counts. To make precise predictions simulations will need to be done.
Recovering the p.d.f.

The Edgeworth expansion

\[
P(\delta)d\delta = \frac{d\delta}{\sqrt{2\pi}\sigma} e^{-\frac{\delta^2}{2\sigma^2}} \left[ 1 + \frac{S_3\sigma}{3!} H_3 \left( \frac{\delta}{\sigma} \right) + \frac{1}{2} \left( \frac{S_3\sigma}{3!} \right)^2 H_6 \left( \frac{\delta}{\sigma} \right) + \frac{S_4\sigma^2}{4!} H_4 \left( \frac{\delta}{\sigma} \right) + \ldots \right]
\]

Hermite Polynomials

\[
S_n = \frac{\langle \delta^n \rangle_c}{\langle \delta^2 \rangle^{n-1}}
\]
The mass function

Gaussian case (Press–Schechter)–

\[
\frac{dn(M)}{dM} = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} e^{-\frac{\delta_c^2}{2\sigma(M)^2}} \frac{d\ln \sigma}{dM} \frac{\delta_c}{\sigma}
\]

The non–Gaussian mass function

\[
\frac{dn(M)}{dM} = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} e^{-\frac{\delta_c^2}{2\sigma(M)^2}} \left[ \frac{d\ln \sigma}{dM} \left( \frac{\delta_c}{\sigma} + \frac{S_3 \sigma}{6} \left( \frac{\delta_c^4}{\sigma^4} - 2\frac{\delta_c^2}{\sigma^2} - 1 \right) \right) + \frac{1}{6 \frac{dS_3}{dM}} \sigma \left( \frac{\delta_c^2}{\sigma^2} - 1 \right) \right]
\]

\( \delta_c(z) = \) collapse threshold – increases with redshift
Use flat $\Lambda$CDM with
\[ \Omega_m = 0.24 \]
\[ \Omega_b = 0.022/h^2 \]
\[ h = 0.73 \]
\[ \sigma_8 = 0.77 \]
\[ n_s = 0.958 \]
Smooth by top–hat window function
The change in the mass function

\[ f_{NL} = +332 \]
\[ f_{NL} = -256 \]

DBI

dotted \( \kappa = 0 \)
dashed \( \kappa = -0.1 \)
don–dashed \( \kappa = -0.3 \)
Use Sheth–Tormen for Gaussian mass function

assuming $M_{\text{lim}} = 1.75 \times 10^{14} \, h^{-1} \, M_{\odot}$ (ind. of $z$)
$dN/dz$

Use Sheth–Tormen for Gaussian mass function

assuming $M_{\text{lim}} = 1.75 \times 10^{14} h^{-1} M_{\text{sun}}$ (ind. of $z$)
\[ \Delta dN/dz = dN/dz(M_{\text{lim}} + \Delta M) - dN/dz(M_{\text{lim}}) \]
\[ \Delta dN/dz = dN/dz(\sigma_8 + \Delta \sigma_8) - dN/dz(\sigma_8) \]

BUT, the mass limit isn’t perfectly known.

dN/dz

BUT, the mass limit isn’t perfectly known

\[ \Delta dN/dz = dN/dz(M_{lim} + \Delta M) - dN/dz(M_{lim}) \]

\[ \Delta dN/dz = dN/dz(\sigma_8 + \Delta \sigma_8) - dN/dz(\sigma_8) \]
Future Constraints?

WMAP priors + 10% mass uncertainty

Planck Priors + 10% mass uncertainty

(assuming a full sky cluster survey, errors $\propto 1/(f_{\text{sky}})^{1/2}$)
Other methods

The Bispectrum

In principle the bispectrum contains more information because it retains the dependence on $k_1, k_2, k_3$

Unlike the mass function, the bispectrum doesn’t rely on knowledge of higher order cumulants

The bispectrum avoids the issue of expanding the pd.f.
The bispectrum

\[ \langle \delta(k_1)\delta(k_2)\delta(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3)B(k_1, k_2, k_3) \]

\[ B(k_1, k_2, k_3) = B_I(k_1, k_2, k_3) + B_G(k_1, k_2, k_3) \]

due to initial non-Gaussianity

evolved non-Gaussianity due to non-linear evolution
The bispectrum

\[ \langle \delta(k_1)\delta(k_2)\delta(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3)B(k_1, k_2, k_3) \]

\[ B(k_1, k_2, k_3) = B_I(k_1, k_2, k_3) + B_G(k_1, k_2, k_3) \]

\[ Q(k_1, k_2, k_3) = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)} \]
The bispectrum

\[ Q_G(k, k, k) = \frac{4}{7} \]

\[ Q_I(k, k, k) \sim \frac{1}{c_s^2 k^2 T(k)} \sim \frac{k^{-2\kappa}}{k^2 T(k)} \]

dotted \( \kappa = 0 \)

dashed \( \kappa = -0.1 \)

dot-dashed \( \kappa = -0.3 \)

Transfer function

\( T(k/k_{eq} << 1) \sim 1 \)
\( T(k/k_{eq} >> 1) \sim \ln(k)/k^2 \)
The bispectrum

See also Sefusatti & Komatsu astro-ph/07050343
Conclusions

- Models of inflation with $c_s \neq 1$ produce scale dependent non-Gaussianity.
- We have explored possibilities beyond the CMB for putting constraints on this non-Gaussianity.
- Cluster number counts constrain non-Gaussianity at a different scale than CMB and can provide a cross-check to CMB constraints. Simulations will need to be done to make precise constraints.
Slow-roll inflation

\[ \mathcal{L} = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]

\[ \dddot{\phi} + 3H \dot{\phi} + V'(\phi) \]

\[ \epsilon = - \frac{\dot{H}}{H^2} \]

\[ \eta = - \frac{\dot{\epsilon}}{H \epsilon} \]