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Staggered Multi-Field Inflation

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Why are multi-field models interesting?

Review: Wands 07

“Natural” in string theory: many moduli fields are present.

Assisted inflation effect (Liddle, Mazumdar and Schunck 98):

- possible avoidance of super-Planckian field values
- increased Hubble friction, steeper potentials work, reduced fine tuning

Many open issues (good for theorists):

- implementing concrete models in string theory (active field)
- new effects such as staggered inflation
- largely unknown theory or (p)re-heating (danger of heating hidden sectors D. Greene 08, potentially no parametric resonance D. Battefeld, S. Kawai 08; D.Battefeld, T.Battefeld, J.Giblin in preparation)
- possibly large non-Gaussianities (model dependent, see review Wands 07)

Main Question

What are the consequences if fields drop out of multi-field inflation in a staggered fashion?

This is a generic feature in many models of multi-field inflation in string theory.

E.g. in:

- Inflation from multiple tachyons
  Majumdar, Davis 03
- Inflation from multiple M5-branes
  Becker, Becker, Krause 05
- Inflation on the landscape
- …

Outline

1. Concrete examples of multi-field inflation within string theory.

2. Analytic formalism to compute effects due to staggered inflation
   - background (relation to warm inflation)
   - perturbations (adiabatic and isocurvature)
   - observables: scalar spectral index, tensor to scalar ratio, …
   - extensions (several follow up projects possible)

3. Application
   - inflation from tachyons
   - inflation on the landscape
   - comparison with WMAP5

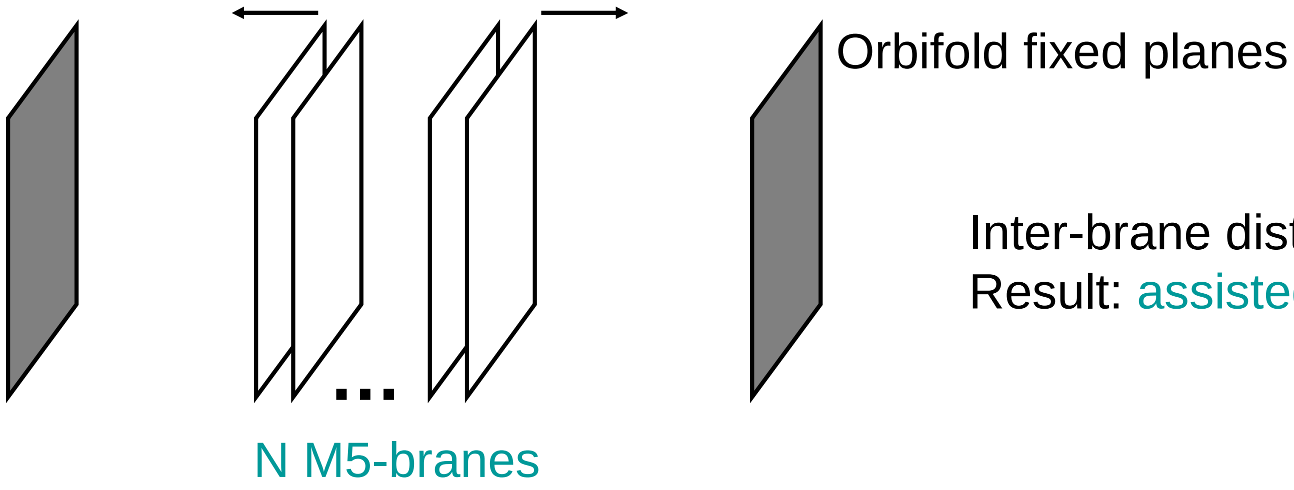
4. Conclusions

Note: $m_p^2 \equiv 1$ throughout

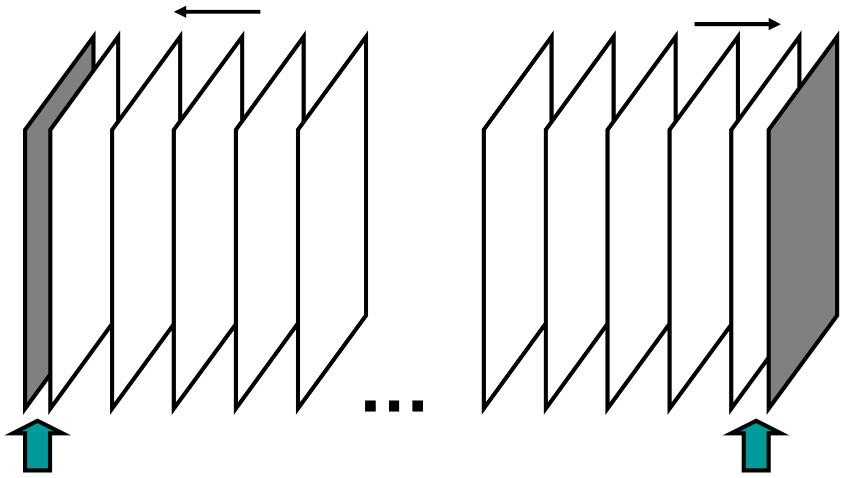
Example: Inflation from M5-branes (BBK setup)

Becker, Becker, Krause 05

Consider Orbifold: $\mathbf{S_1/Z_2}$



Inter-brane distance ⟷ inflatons;
Result: assisted inflation



Branes dissolve into Boundary branes during inflation, one after the other, fields drops out of the model;
energy is converted, e.g. into radiation.

Result: N(t) decreasing during inflation.

Branes collide and dissolve, one after the other: staggered inflation (cascade inflation).

Numerical treatment:
Ashoorioon, Krause 06, &Turzynski 08

Example: Inflation from multiple tachyons   Majumdar, Davis 03

N D-brane/antibrane pairs give rise to U(N)xU(N) sym.:
N^2 coupled tachyons,

M.&D. focus on abelian part:
N, uncoupled fields.

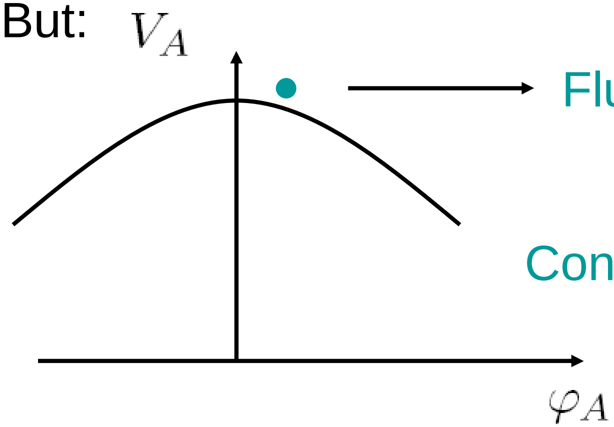
Result: assisted inflation

Potential (trustworthy near origin):

$$W = \sum_{A=1}^{\mathcal{N}} V_A$$
$$= \mathcal{N}\tau_p - c_1 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^2 + c_2 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^4 + \mathcal{O}\left(|\varphi_A|^6\right)$$

Review: K. Ohmori 01 $c_1 \approx 0.87$ $c_2 \approx 0.21$

But:



Fluctuation (e.g. QM)

↓

Condensation (sudden): field drops out of model,
its energy is converted, e.g. into radiation.

One field after the other drops out, N(t) decreasing during inflation:
Staggered Inflation

Analytic Formalism to deal with fields dropping out:

Smooth out N(t) and introduce a continuous decay rate:

$$\Gamma(t) \equiv -\dot{\mathcal{N}}/\mathcal{N}.$$

The rate is model dependent and can depend on time. For this smoothing to be a good approximation, we need that in any given Hubble time of interest several fields drop out.

Further simplifying assumptions (not crucial):
- uncoupled fields
- identical potentials (relaxed later on)
- identical initial conditions (relaxed later on)

Effects we recover:
- additional decrease of energy that drives inflation
- additional, leading order contributions to observables (e.g. scalar spectral index)

Effects we do not recover:
- sharp steps in observables such as the scalar spectral index
- ringing in the spectrum
- additional grav.waves, non-Gaussianities caused by decaying fields

Background evolution

Introduce effective single field: $\varphi \equiv \sqrt{\mathcal{N}}\varphi_A$

$$W(\varphi) = \mathcal{N} V(\varphi/\sqrt{\mathcal{N}})$$

Energy transfer to additional component:

$$\dot{\rho}_\varphi = -3H(\rho_\varphi + p_\varphi) + \dot{\mathcal{N}} V$$
$$\dot{\rho}_r = -3H(\rho_r + p_r) - \dot{\mathcal{N}} V$$

See Watson, Perry, Kane, Adams 06 for related work on a relaxing CC.

Introduce small parameters:

$$\bar{\varepsilon} \equiv \frac{3}{2}(1+w_r)\frac{\rho_r}{\rho_\varphi + \rho_r}$$

$$\varepsilon_{\mathcal{N}} \equiv -\frac{\dot{\mathcal{N}}}{\mathcal{N}}\frac{1}{2H} = \frac{\Gamma}{2H}$$

$$\hat{\varepsilon} \equiv -\frac{\dot{H}}{H^2}$$

$$\simeq \varepsilon + \bar{\varepsilon}$$

$$\varepsilon_A \equiv \frac{1}{2}\left(\frac{V_A'}{W}\right)^2 \ll 1 \quad , \quad \varepsilon \equiv \frac{1}{2}\left(\frac{W'}{W}\right)^2 \ll 1 \, ,$$

$$\eta_A \equiv \frac{V_A''}{W} \quad , \quad |\eta_A| \ll 1 \, ,$$

$$\eta \equiv \frac{W''}{W} \quad , \quad |\eta| \ll 1 \, ,$$

Can show that within inflationary models of interest to first order in small parameters:

$$\varepsilon_{\mathcal{N}} \simeq \bar{\varepsilon}$$

Background evolution

We get

$$\begin{aligned} \dot{\rho}_\varphi &\simeq -2H(\varepsilon_{\mathcal{N}} + \varepsilon)\rho_\varphi \\ \dot{\rho}_r &\simeq 2H(\varepsilon_{\mathcal{N}} - \bar{\varepsilon})\rho_\varphi \end{aligned}$$

This leads to a scaling solution during inflation.

Then the effective single field evolves according to

$$\begin{aligned} 3H\dot{\varphi} &\simeq -W'\gamma \\ \gamma &\equiv 1 + \varepsilon_{\mathcal{N}}\varphi\frac{W}{W'} \end{aligned}$$

Resembles warm Inflation (Barera 95),
but

- the radiation-bath originates from transferring potential energy, not kinetic
- model can arise in string theory
- avoids many problems of regular warm inflation

Next, perturbations; new effects due to

- presence of ρ_r and perturbations therein
- additional decrease of W due to the decay rate $\Gamma \neq 0$

Perturbations (straightforward)

We can show that isocurvature/entropy perturbations are suppressed (follow Malik, Wands, Ungarelli 03, …), so we can focus on adiabatic perturbations.

Use the Mukhanov variable, satisfying $$v_k'' + \left(k^2 c_s^2 - \frac{z''}{z}\right) v_k = 0$$

Where $$z \equiv \frac{1}{\theta c_s}, \quad \theta^2 \equiv \frac{1}{3a^2(1+w)} \qquad w = p/\rho$$

$$p = p_\varphi + p_r \qquad \rho = \rho_\varphi + \rho_r$$

Focus on large scales, so

$$c_s^2 \approx \dot{p}/\dot{\rho}$$

Imposing QM initial conditions and using the background solutions, we can compute the curvature perturbation

$$v_k = z\zeta_k$$

and the power-spectrum: $$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2}|\zeta_k|^2$$

Perturbations

The scalar spectral index $$n_s - 1 = \frac{d \ln \mathcal{P}_\zeta}{d \ln k}$$

becomes

$$n_s - 1 \simeq -2(\varepsilon + \varepsilon_{\mathcal{N}}) - \frac{2}{\varepsilon\gamma^2 + \varepsilon_{\mathcal{N}}}\left[\varepsilon\gamma^2(2\varepsilon - \varepsilon_{\mathcal{N}} - \eta) + (\varepsilon + \varepsilon_{\mathcal{N}})(1-\delta)(\varepsilon\gamma(\gamma - 1) + \frac{\varepsilon_{\mathcal{N}}}{2})\right]$$

Recover known limits:

$$\delta \equiv \frac{\dot{\Gamma} H}{\Gamma \dot{H}}$$

- Usual slow roll: $\left.\begin{array}{l}\Gamma = 0 \\ \varepsilon_{\mathcal{N}} = 0\end{array}\right\}$ $n_s^{SR} - 1 \simeq -6\varepsilon + 2\eta$

- Dynamically relaxing CC (Inflation without Inflatons):

$$\left.\begin{array}{l}\varepsilon_{\mathcal{N}} = const \\ \delta = 1 \\ \varepsilon = \eta = 0\end{array}\right\} \quad n_s^{relax.CC} - 1 = -2\varepsilon_{\mathcal{N}}$$

Watson, Perry, Kane, Adams 06 .

Perturbations

If slow roll contributions are negligible (most interesting case) we get

$$\mathcal{P}_\zeta \simeq \frac{1}{8\pi^2\bar{\varepsilon}}\frac{H^2}{m_{pl}^2}$$
$$n_s - 1 \simeq (\delta - 3)\bar{\varepsilon}$$

$$\frac{\Gamma}{2H} \equiv \varepsilon_{\mathcal{N}} \simeq \bar{\varepsilon} \equiv \frac{3}{2}(1+w_r)\frac{\rho_r}{\rho_r+\rho_I}$$
$$\delta \equiv \frac{\dot{\Gamma}H}{\Gamma\dot{H}}$$

A similar computation for gravity waves leads to

$$\mathcal{P}_T \simeq \frac{2}{\pi^2}\frac{H^2}{m_{pl}^2}$$
$$n_T \equiv \frac{d\,\ln\mathcal{P}_T}{d\,\ln k} \simeq -2\bar{\varepsilon}\,.$$

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \simeq 16\bar{\varepsilon}\,.$$

Summary of the Formalism

The expressions are general, but rely on the smoothing (time averaging) of $N(t)$, assuming slow roll and for simplicity identical, uncoupled field potentials and identical initial field values (the latter two we will relax later on).

Still to do:

Check the validity of analytic framework by comparing with numerics.

Main Question: how large do N and Gamma have to be?

Application 1: inflation from tachyons

Majumdar, Davis 03
Battefeld, Battefeld, Davis 08

Consider

$$W = \mathcal{N}\tau_p - c_1 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^2 + c_2 \sum_{A=1}^{\mathcal{N}} |\varphi_A|^4 + \mathcal{O}\left(|\varphi_A|^6\right)$$

$$c_1 \approx 0.87$$

$$c_2 \approx 0.21$$

Cases:

- Exponential decrease of N:

$$\Gamma = const$$

- Serial condensation: $\mathcal{N}(t) = \mathcal{N}_0(1 - t/\tilde{t})$

$$\Gamma = 1/(\tilde{t} - t)$$

- Condensation all at once (unlikely):

$$\Gamma = 0$$

Discuss these cases only, assuming negligible slow roll contributions, that is assuming that the fields are very close to the origin – this is actually the best motivated case.

(for inclusion of slow roll contributions and application to different setups, see arxiv:0806.1953)

Application 1: inflation from tachyons

1. Exponential condensation: $\Gamma = const$

$$\mathcal{N}(t) = \mathcal{N}_0 e^{-\Gamma t}$$

We need around N=60 e-folds of inflation: $N = \int_{t_{ini}}^{t_{end}} H\, dt$

Inflation ends once $\mathcal{N} \sim 1$

so that $N \approx \frac{2}{\Gamma}\left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2}\left(1 - \frac{1}{\sqrt{\mathcal{N}_0}}\right) \approx \frac{2}{\Gamma}\left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2}$

Further $\varepsilon_{\mathcal{N}} = \frac{\Gamma}{2}\left(\frac{3}{\tau_p \mathcal{N}_0}\right)^{1/2} \approx \frac{1}{N}$

Applying the general formula for the scalar spectral index:

$$\boxed{n_s - 1 \simeq -3\varepsilon_{\mathcal{N}} \approx -\frac{3}{N}}$$

If we set $\tau_p \equiv c_1^2/(4c_2) \approx 0.90$ we need $\frac{\mathcal{N}_0}{\Gamma^2} \approx \frac{3N^2 c_2}{c_1^2} \approx 3000$

Application 1: inflation from tachyons

2. serial condensation: $\Gamma = (\tilde{t} - t)^{-1}$

$$\mathcal{N}(t) = \mathcal{N}_0 \left(1 - \frac{t}{\tilde{t}}\right)$$

We need around 60 e-folds of inflation: $N = \int_{t_{ini}}^{t_{end}} H \, dt$

Inflation ends once $\mathcal{N} \sim 1$

so that $N \approx \frac{2\tilde{t}}{3} \left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2} \left(1 - \frac{1}{\mathcal{N}_0^3}\right) \approx \frac{2\tilde{t}}{3} \left(\frac{\tau_p \mathcal{N}_0}{3}\right)^{1/2}$

Further $\varepsilon_{\mathcal{N}} \approx 1/(3N)$

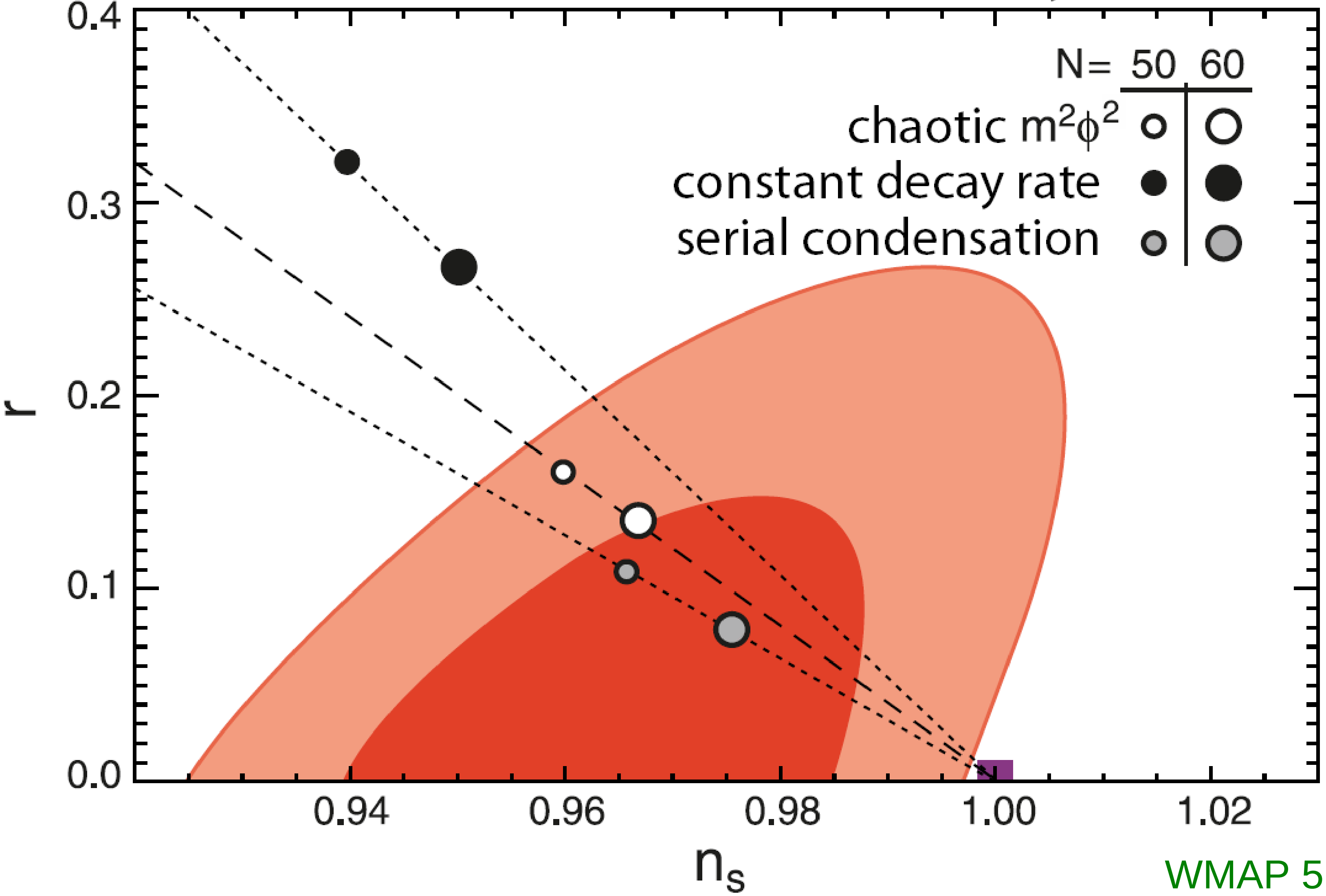
$$\delta \simeq -2\varepsilon_{\mathcal{N}}/\hat{\varepsilon} \simeq -2$$

Applying the general formula for the scalar spectral index:

$$\boxed{n_s - 1 \simeq -5\varepsilon_{\mathcal{N}} \approx -\frac{5}{3N}}$$

If we set $\tau_p \equiv c_1^2/(4c_2) \approx 0.90$ we need $\mathcal{N}_0 \tilde{t}^2 \approx \frac{27 N^2 c_2}{c_1^2} \approx 27000$

Staggered Inflation from Tachyons



WMAP 5

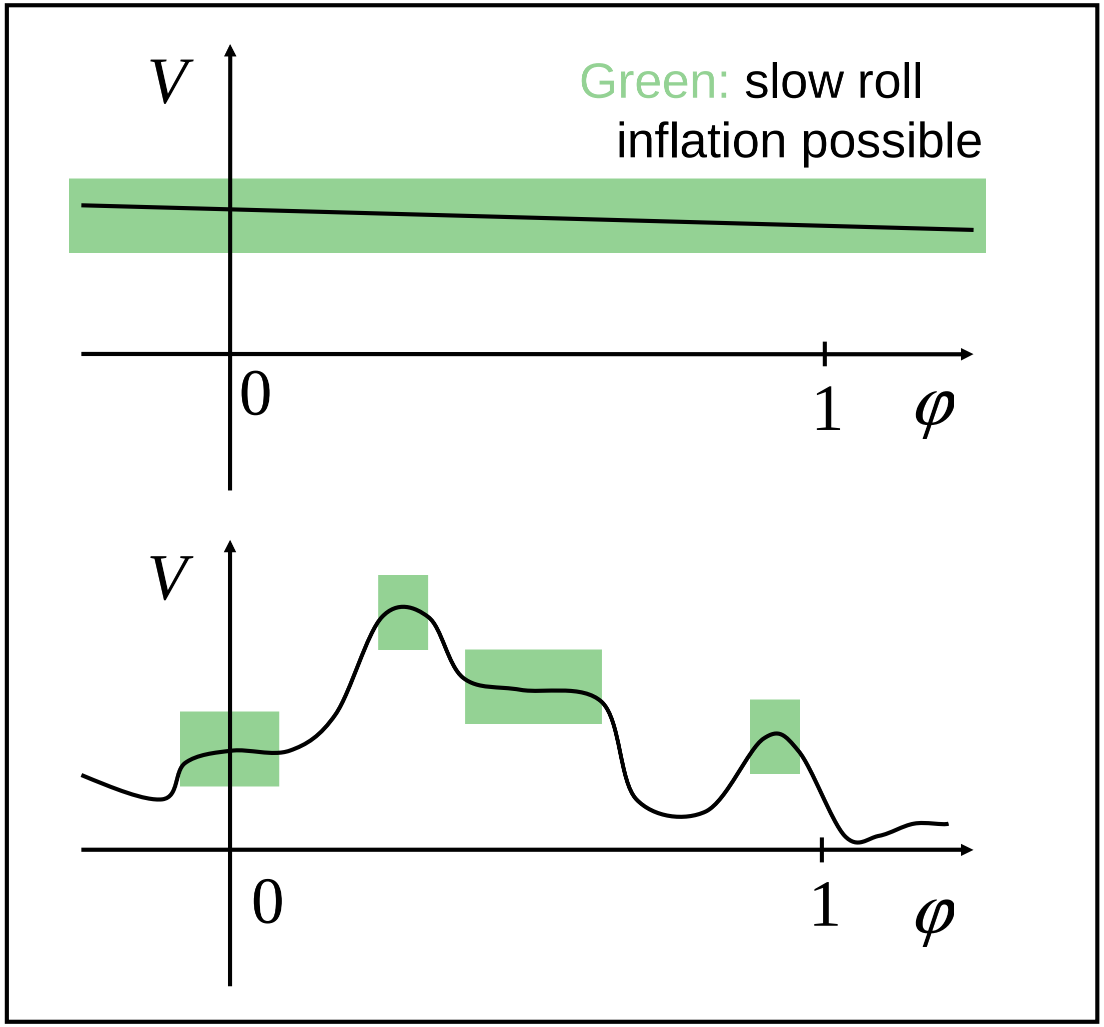
Application 2: Inflation on the Landscape

D.Battefeld, T.Battefeld work in progress.

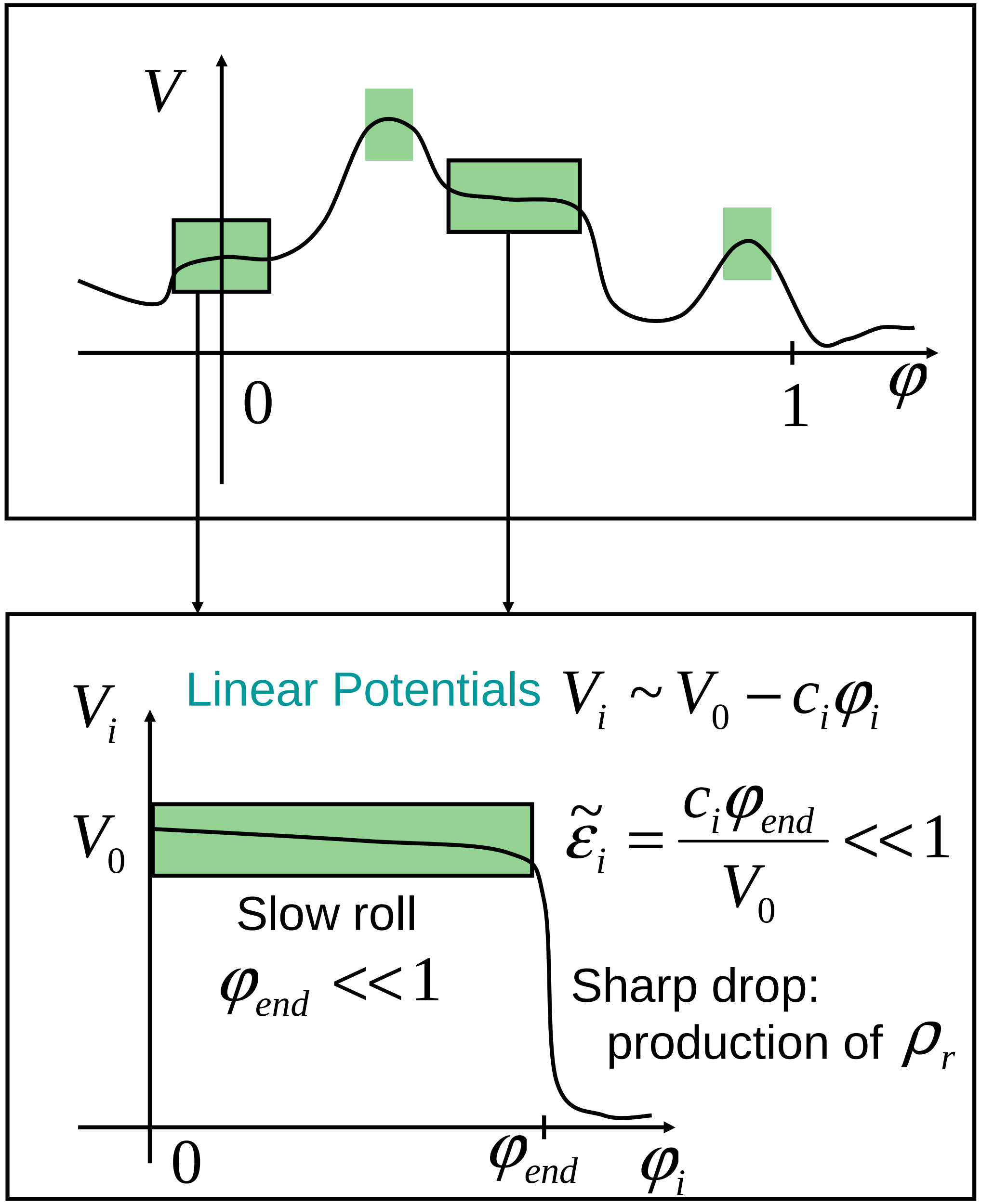
What potentials do we expect from string theory?

A: Long flat stretches? (needed for single field inflation).

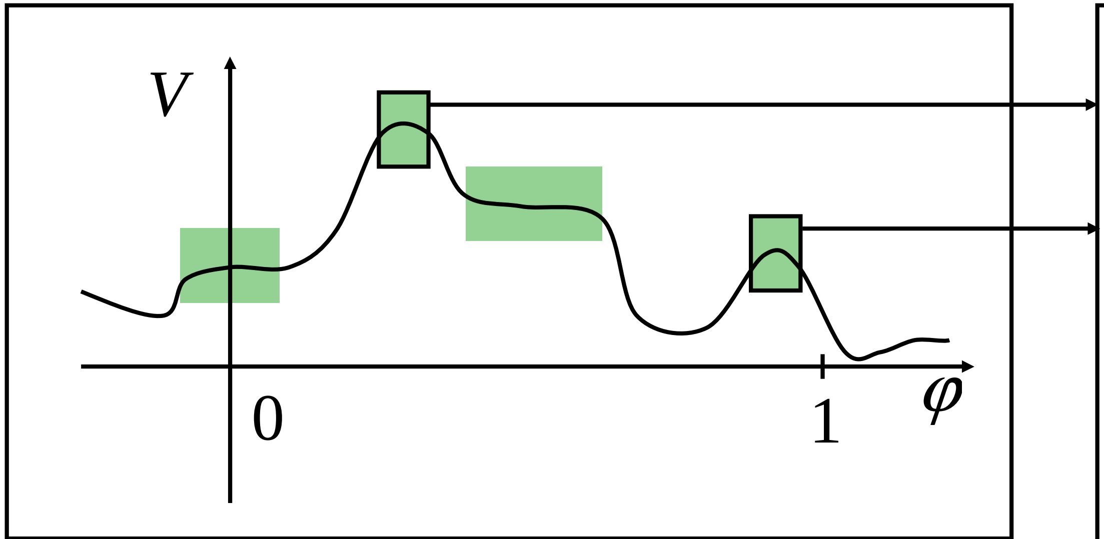
B: Short flat stretches, hills and valleys, sharp drops, …



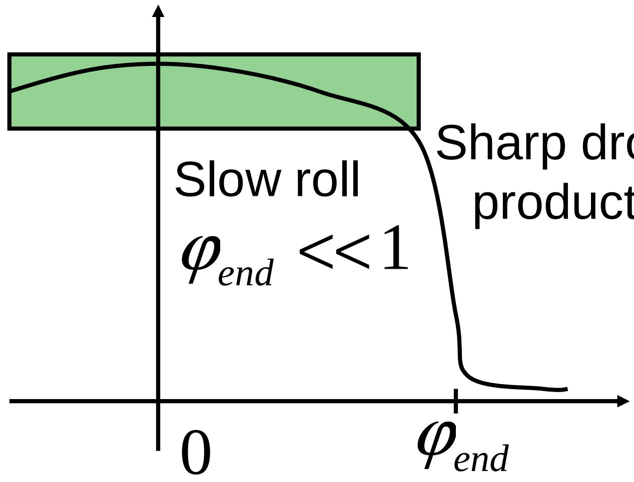
Expand the Potential around a flat stretch:



Expand the Potential around a flat stretch:



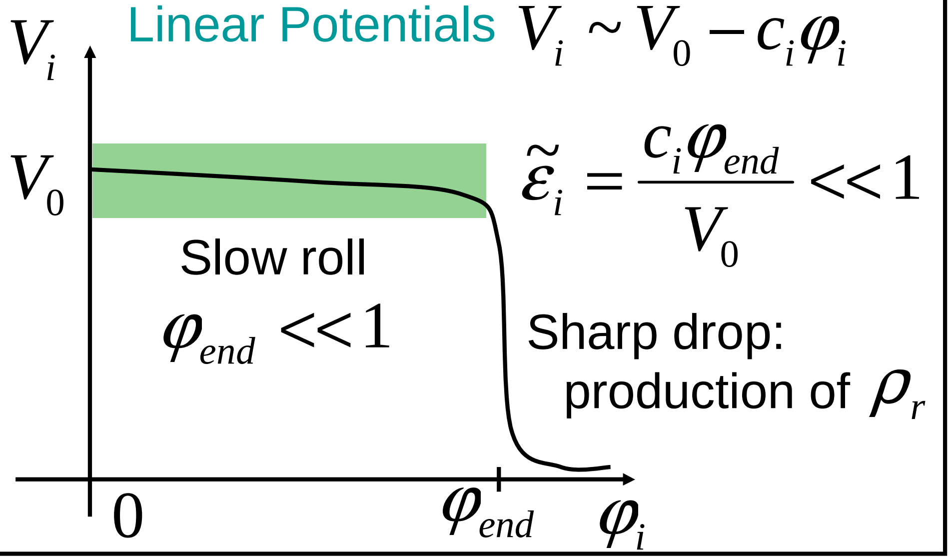
Hilltop Potentials



$$V_i \sim V_0 - \frac{m_i^2}{2}\varphi_i^2$$

$$\tilde{\varepsilon}_i = \frac{m_i^2 \varphi_{end}^2}{V_0} << 1$$

Linear Potentials $V_i \sim V_0 - c_i \varphi_i$



$$\tilde{\varepsilon}_i = \frac{c_i \varphi_{end}}{V_0} << 1$$

In either case we arrive at staggered inflation, but the decay rate is not a free parameter.

Initial conditions and potentials:

Spread fields evenly over the allowed domain:

$$\varphi_i(t=0) = \varphi_{ini} + \frac{\varphi_{end} - \varphi_{ini}}{\mathcal{N}(t=0)}(i-1)$$

Potentials: allow for slightly different slopes or masses: $l \ll 1$

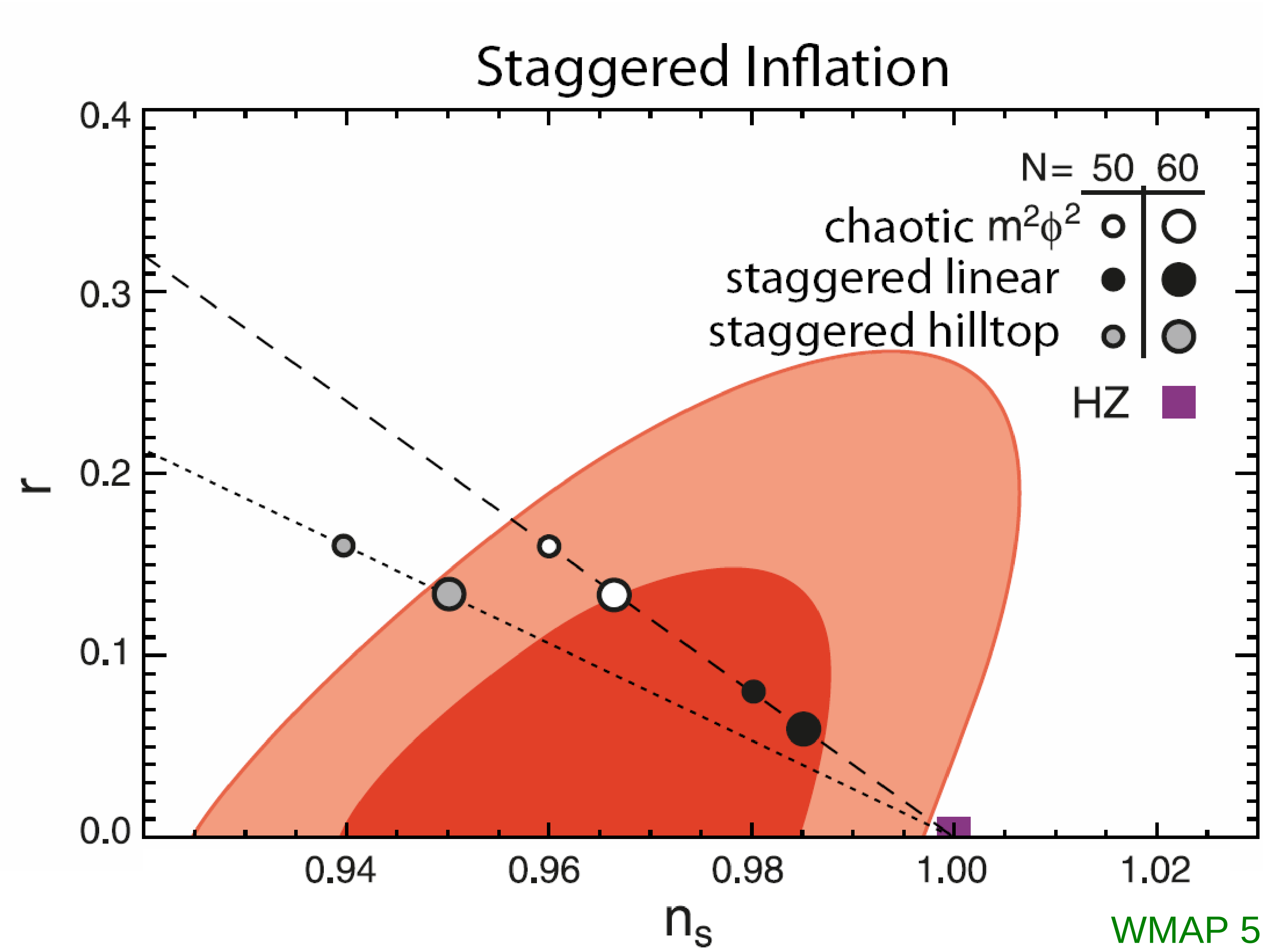
Linear: $c_i = c\left(1 + l\frac{i}{\mathcal{N}}\right)$

Hilltop: $m_i^2 = m^2(1 + li/\mathcal{N})$

- solve slow roll equations of motion
- compute decay rate, delta and number of e-folds
- compute observables

$$\begin{aligned} n_s - 1 &\simeq -\frac{1}{N}\left(1 + \frac{2l}{3}\right), \\ n_T &\simeq -\frac{1}{2N}\left(1 + \frac{2l}{3}\right), \\ r &\simeq \frac{4}{N}\left(1 + \frac{2l}{3}\right). \end{aligned}$$

$$\begin{aligned} n_s - 1 &\approx -\frac{3}{N}\left(1 + l\frac{3}{4}\right), \\ n_T &\approx -\frac{1}{N}\left(1 + l\frac{3}{4}\right), \\ r &\approx \frac{8}{N}\left(1 + l\frac{3}{4}\right). \end{aligned}$$


Staggered Inflation
0.4
0.3
0.2
0.1
0.0
r
0.94
0.96
0.98
1.00
1.02
n_s
N= 50 60
chaotic m²φ²
staggered linear
staggered hilltop
HZ
WMAP 5

Other unique signatures?

Whenever fields decay, slow roll is violated and we can get:

- additional Gravitational waves (similar to GW from reheating, see e.g. Easther, Giblin, Lim, 06, 07, 08);

- Non-Gaussianities (similar to NG from steps in a potential, see Easther, Chen, Lim 06, 08)

To compute either of them, the decay of fields needs to be understood better.

Need: a concrete implementation within string theory to make progress.

Conclusions

Staggered inflation occurs naturally in several multi-field models within string theory.

We developed an analytic formalism to compute effects on some observables (scalar spectral index, running, gravity waves, …).

Many possible follow up projects

- compute additional gravitational waves, Non-Gaussianities, …

- revisit other existing models

- compare with full numerical studies to check validity of the analytic framework

- construct concrete models based on staggered inflation effect within string theory

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